A brief survey of post-quantum cryptography

D. J. Bernstein
University of Illinois at Chicago

"Once the enormous
energy boost that quantum
computers are expected
to provide hits the street,
most encryption security
standards—and any
other standard based on
computational difficulty—
will fall, experts believe."

(Magiq's web site, 2008;
the "experts" aren't named)

Is cryptography dead?

Imagine:
15 years from now
someone announces
successful construction
of a large quantum computer.

New York Times headline:
"INTERNET CRYPTOGRAPHY KILLED BY PHYSICISTS."

Users panic.

What happens to cryptography?

RSA: Dead.

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DSA: Dead.

ECDSA: Dead.

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ECC in general: Dead.

HECC in general: Dead.

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Buchmann–Williams: Dead.

Class groups in general: Dead.

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“They’re all dead, Dave.”

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Class groups in general: Dead.

"They're all dead, Dave."

But we have other types of cryptographic systems!

Hash-based cryptography.

Example: 1979 Merkle hash-tree public-key signature system.

Code-based cryptography.
Example: 1978 McEliece
hidden-Goppa-code
public-key encryption system.

Lattice-based cryptography.
Example: 1998 "NTRU."

Multivariate-quadratic-equations cryptography.
Example:
1996 Patarin "HFE^{v-}"
public-key signature system.

Secret-key cryptography.
Example: 1998 Daemen–Rijmen
"Rijndael" cipher, aka "AES."

Nobody has figured out
a way to apply Shor's algorithm
to any of these systems.

Grover's algorithm
does have some applications,
but cryptographers
can easily compensate
by scaling up somewhat.

“Maybe there’s a better attack!”

This was already a familiar risk
before quantum computers.

This is why the community
puts tremendous effort
into cryptanalysis.

Results of cryptanalysis:
Some systems are killed.
Some systems need larger keys.
Some systems inspire confidence.

1978 RSA paper mentions Schroeppel's "linear sieve" for factorization. Need $b^{2+o(1)}$-bit RSA key for b-bit security.

1988 Pollard et al.: "Number-field sieve." Need $b^{3+o(1)}$-bit RSA key for b-bit security.

Many improvements since then, but still $b^{3+o(1)}$ ... against classical computers.

1994 Shor: Need intolerably large RSA key for b-bit security against quantum computers.

1978 McEliece paper
mentions a decoding algorithm.
Need code dimension
$b^{1+o(1)}$ for b-bit security.

Many improvements since then,
but still dimension $b^{1+o(1)}$
against classical computers.

Still dimension $b^{1+o(1)}$
against quantum computers.

If McEliece security is so good,
why are we still using RSA?
Answer: McEliece key is huge.

Pre-quantum cryptography:

Cryptographers design systems to scramble and unscramble data.
RSA, McEliece, AES, many more.

Cryptanalysts break some systems using $< 2^b$ classical operations.
Tools: NFS, LLL, F4, etc.

Unbroken systems:
RSA with $b^{3+o(1)}$ bits,
McEliece with dimension $b^{1+o(1)}$,
AES if $b \leq 256$, etc.

Algorithm designers and implementors find the fastest unbroken systems.

Post-quantum cryptography:

Cryptographers design systems to scramble and unscramble data.
RSA, McEliece, AES, many more.

Cryptanalysts break some systems using $< 2^b$ quantum operations.
Tools: NFS, LLL, F4, etc.
plus Shor, Grover, etc.

Unbroken systems:
McEliece with dimension $b^{1+o(1)}$,
AES if $b \leq 128$, etc.

Algorithm designers and implementors find the fastest unbroken systems.

A hash-based signature system

Standardize a 256-bit
hash function H.

Signer's public key: 512 strings
$y_1[0], y_1[1], \ldots, y_{256}[0], y_{256}[1]$,
each 256 bits.
Total: 131072 bits.

Signature of a message m:
256-bit strings $r, x_1, \ldots, x_{256}$
such that the bits $(h_1, \ldots, h_{256})$
of $H(r, m)$ satisfy
$y_1[h_1] = H(x_1)$, $\ldots$,
$y_{256}[h_{256}] = H(x_{256})$.

Signer's secret key:
512 independent uniform
random 256-bit strings
$x_1[0], x_1[1], \ldots, x_{256}[0], x_{256}[1]$.

Signer computes
$y_1[0], y_1[1], \ldots, y_{256}[0], y_{256}[1]$
as $H(x_1[0]), H(x_1[1]), \ldots,$
$H(x_{256}[0]), H(x_{256}[1])$.

To sign m:
generate uniform random r;
$H(r, m) = (h_1, \ldots, h_{256})$;
reveal $(r, x_1[h_1], \ldots, x_{256}[h_{256}])$;
discard remaining x values;
refuse to sign more messages.

This is the "Lamport–Diffie one-time signature system."

How to sign
more than one message?

Easy answer: "Chaining."
Signer expands m to include a newly generated public key that will sign next message.

More advanced answers
(Merkle et al.)
scale logarithmically with the number of messages signed.

A code-based encryption system

Receiver's public key:
1800×3600 bit matrix K.

Messages suitable for encryption:
3600-bit strings of "weight 150";
i.e., 3600-bit strings
with exactly 150 nonzero bits.

Encryption of m
is 1800-bit string Km.

Attacker, by linear algebra,
can easily work backwards
from Km to *some* v
such that $Kv = Km$.

Huge number of choices of v.
Finding weight-150 choice
("syndrome-decoding K")
seems extremely difficult
for most choices of K.

Best attacks? See next talk.

Receiver secretly generates public key K with a "hidden Goppa code" structure that allows fast decoding.

Detecting this structure seems even more difficult than syndrome-decoding random K.

Receiver starts with secret monic degree-150 irreducible polynomial $g \in \mathbf{F}_{4096}[x]$ and distinct $\alpha_1, \ldots, \alpha_{3600} \in \mathbf{F}_{4096}$.

"Patterson's algorithm" syndrome-decodes the matrix

$$H = \begin{pmatrix} \frac{1}{g(\alpha_1)} & \cdots & \frac{1}{g(\alpha_{3600})} \\ \frac{\alpha_1}{g(\alpha_1)} & \cdots & \frac{\alpha_{3600}}{g(\alpha_{3600})} \\ \vdots & \ddots & \vdots \\ \frac{\alpha_1^{149}}{g(\alpha_1)} & \cdots & \frac{\alpha_{3600}^{149}}{g(\alpha_{3600})} \end{pmatrix}.$$

Receiver also has
a secret invertible
1800×1800 matrix S
and a secret 3600×3600
permutation matrix P.

Receiver's public key K
is the product SHP.

Given ciphertext $Km = SHPm$:
receiver computes HPm;
decodes H to obtain Pm;
computes m.

This is 1986 Niederreiter variant of McEliece's original system.

Reducing K to "systematic form" reduces space to 3211248 bits.
Many other improvements.

Lattice-based cryptography:
similar; more complicated;
maybe more attractive!
See tomorrow's talk.

An MQ signature system

Signer's public key:
polynomials $P_1, \ldots, P_{300}$
$\in \mathbf{F}_2[w_1, \ldots, w_{600}]$.

Extra requirements
on each of these polynomials:
degree ≤ 2, no squares;
i.e., linear combination of
$1, w_1, \ldots, w_{600}$,
$w_1w_2, w_1w_3, \ldots, w_{599}w_{600}$.

Overall 54090300 bits.

Signature of m:
a 300-bit string r and
values $w_1, \ldots, w_{600} \in \mathbf{F}_2$
such that $H(r, m) =$
$(P_1(w_1, \ldots, w_{600}), \ldots,$
$P_{300}(w_1, \ldots, w_{600}))$.

Only 900 bits!

Verifying a signature uses
one evaluation of H and
millions of bit operations
to evaluate $P_1, \ldots, P_{300}$.

Main challenge for attacker:
find bits $w_1, \ldots, w_{600}$
producing specified outputs
$(P_1(w_1, \ldots, w_{600}), \ldots,$
$P_{300}(w_1, \ldots, w_{600}))$.

Random guess: on average,
only 2^{-300} chance of success.

"XL" etc.: fewer operations,
but still not practical.

Signer generates public key
with secret "HFE$^{\mathrm{v}-}$" structure.

Standardize a degree-450
irreducible polynomial $\varphi \in \mathbf{F}_2[t]$.
Define $L = \mathbf{F}_2[t]/\varphi$.

Critical step in signing:
finding roots of a
secret polynomial in $L[x]$
of degree at most 300.

Secret polynomial is chosen with all nonzero exponents of the form $2^i + 2^j$ or 2^i. (So degree ≤ 288.)

If $x_0, x_1, \ldots, x_{449} \in \mathbf{F}_2$ and $x = x_0 + x_1 t + \cdots + x_{449} t^{449}$ then $x^2 = x_0 + x_1 t^2 + \cdots + x_{449} t^{898}$, $x^4 = x_0 + x_1 t^4 + \cdots + x_{449} t^{1796}$, etc.

In general, $x^{2^i+2^j}$
is a quadratic polynomial
in the variables $x_0, \ldots, x_{449}$.

Signer's secret key:

invertible 600×600 matrix S;

300×450 matrix T of rank 300;

$Q \in L[x, v_1, v_2, \ldots, v_{150}]$.

Each term in Q

has one of the forms

$\ell x^{2^i+2^j}$ with $\ell \in L$, $2^i < 2^j$,

$2^i + 2^j \leq 300$;

$\ell x^{2^i} v_j$ with $\ell \in L$, $2^i \leq 300$;

$\ell v_i v_j$;

ℓx^{2^i};

ℓv_j;

ℓ.

To compute public key:

Compute $S(w_1, \ldots, w_{600}) =$
$(x_0, \ldots, x_{449}, v_1, \ldots, v_{150})$.

In $L[w_1, \ldots, w_{600}]$
compute $x = \sum x_i t^i$
and $y = Q(x, v_1, v_2, \ldots, v_{150})$
modulo $w_1^2 - w_1, \ldots, w_{600}^2 - w_{600}$.

Write $y = y_0 + \cdots + y_{449} t^{449}$
with $y_i \in \mathbf{F}_2[w_1, \ldots, w_{600}]$.

Compute $(P_1, \ldots, P_{300}) =$
$T(y_0, y_1, \ldots, y_{449})$.

Sign by working backwards.

Given values $(P_1, \ldots, P_{300})$, invert T to obtain values $(y_0, \ldots, y_{449})$. 2^{150} choices; randomize.

Choose $(v_1, \ldots, v_{150})$ randomly. Substitute into $Q(x, v_1, \ldots, v_{150})$ to obtain $Q(x) \in L[x]$.

Solve $Q(x) = y$ for $x \in L$.
If several roots, randomize.
If no roots, start over.

Invert S to obtain signature.

This is an "HFE$^{\mathrm{v}-}$" example.

"HFE": "Hidden Field Equation"
$Q(x) = y$.

"$-$": publish only 300 equations
instead of 450.

"v": "vinegar" variables
$v_1, \ldots, v_{150}$.

State-of-the-art attack
breaks a simplified system with
0 vinegar variables, 1 term in Q.

Can build MQ systems
in many other ways.

Preparing for the future

When someone announces
a large quantum computer
we'll switch to McEliece etc.

Why worry about the switch
now?

Answer 1: We need time
to improve efficiency.

Answer 2: We need time
to build confidence.

Answer 3: We need time
to improve usability.

Have you implemented
a public-key system?

Send your software to eBATS:
ECRYPT Benchmarking of
Asymmetric Systems.

Now integrated into eBACS:
ECRYPT Benchmarking of
Cryptographic Systems.

`http://bench.cr.yp.to`

2003.09 Bernstein, `sci.crypt`:
“I’m thinking about
publishing a paper
on post-quantum cryptography.
This isn’t too early
to start planning ahead
for the very real possibility
of quantum computers.”

2004.10 Buchmann et al.,
“Post-Quantum Signatures”:
“We would like to thank
Dan Bernstein for inventing
the notion ‘post-quantum
cryptography.’ ”

2004.10 Silverberg to Bernstein:
Hey, let's run a workshop.

2004.10 Bernstein to Silverberg:
How about a workshop on
post-quantum cryptography?

Independently, late 2004:
Wolf proposes workshop on
post-quantum cryptography.

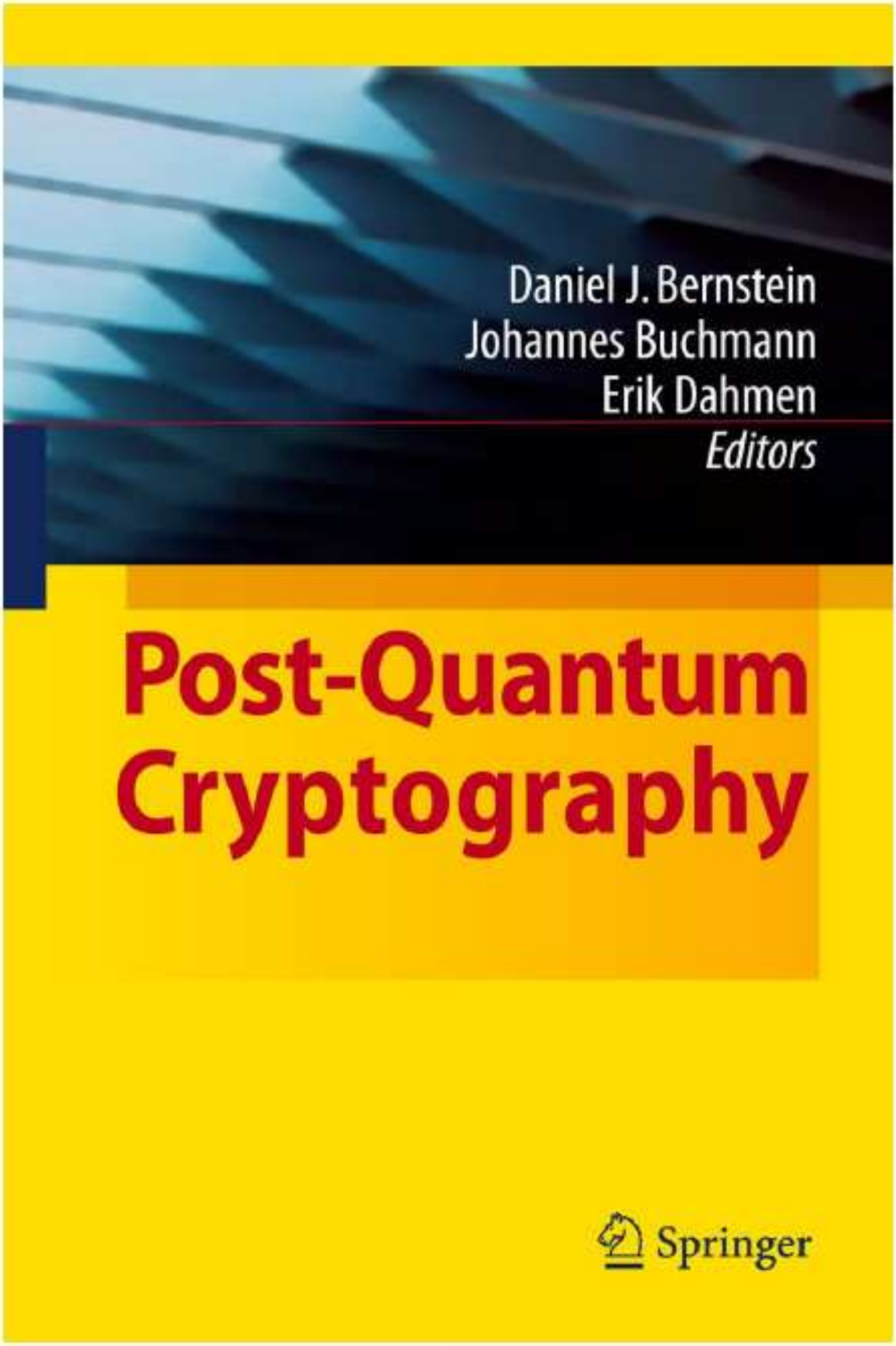
Independently, late 2004:
Ding proposes workshop on
"Cryptology in the
quantum computer era."

2005.05 Lange manages to herd Bernstein, Ding, Nguyen, Wolf, et al.; ECRYPT starts organizing PQCrypto 2006.

2006.05: PQCrypto 2006 in Leuven, Belgium.

2008.10: PQCrypto 2008 here in Cincinnati.

End of 2008: Survey book!


Daniel J. Bernstein
Johannes Buchmann
Erik Dahmen
Editors
Post-Quantum Cryptography
Springer

Bernstein: "Introduction to post-quantum cryptography."

Hallgren, Vollmer: "Quantum computing."

Buchmann, Dahmen, Szydlo: "Hash-based digital signature schemes."

Overbeck, Sendrier: "Code-based cryptography."

Micciancio, Regev: "Lattice-based cryptography."

Ding, Yang: "Multivariate public key cryptography."