Lattice-based public-key cryptosystems

D. J. Bernstein

NIST post-quantum competition:
69 submissions in first round,
from hundreds of people.
(+13 submissions that NIST
declared incomplete or improper.)

22 signature-system submissions.
5 lattice-based: Dilithium;
DRS (broken); FALCON*;
pqNTRUSign*; qTESLA.

47 encryption-system submissions.
20 lattice-based: Compact LWE*
(broken); Ding*; EMBLEM;
Frodo; HILA5 (CCA broken);
KCL*; KINDI; Kyber; LAC; LIMA;
Lizard*; LOTUS; NewHope;
NTRUEncrypt; NTRU HRSS;
NTRU Prime; Odd Manhattan;
Round2*; SABER; Titanium.

*: submitter claims patent on
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Let's try

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Debian: `apt inst`
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Source: `www.sage`
Web: `sagecell.s`

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Source: www.sagemath.org

Web: sagecell.sagemath

Sage is Python 2
+ many math libraries
+ a few syntax differences:

```
sage: 10^6 # power, not x
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sage: factor(31415926535
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```
sage: 10^6 # power, not xor
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4*x^2 + x + 3
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sage: g
x^2 + 7*x + 2
sage: f+g    # built-in add
5*x^2 + 8*x + 5
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5*x^2 + 8*x + 5
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5*x^2 + 8*x + 5
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4*x^3 + x^2 + 3*x
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4*x^4 + x^3 + 3*x^2
sage: f*2
8*x^2 + 2*x + 6
sage: f*(7*x)
28*x^3 + 7*x^2 + 21*x
sage:
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sage: f*(7*x)
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4*x^4 + 29*x^3 + 18*x^2 + 23*x
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8*x^2 + 2*x + 6
sage: f*(7*x)
28*x^3 + 7*x^2 + 21*x
sage: f*g
4*x^4 + 29*x^3 + 18*x^2 + 23*x
 + 6
sage: f*g == f*2+f*(7*x)+f*x^2
True
sage:
```

```
x.<x> = ZZ[]
 now Zx is a class
 Zx objects are polys
 in x with int coeffs
 = Zx([3,1,4])


 x + 3
 = Zx([2,7,1])
```

```
*x + 2
+g    # built-in add
 8*x + 5
```



```
sage: f*x   # built-in mul
4*x^3 + x^2 + 3*x
sage: f*x^2
4*x^4 + x^3 + 3*x^2
sage: f*2
8*x^2 + 2*x + 6
sage: f*(7*x)
28*x^3 + 7*x^2 + 21*x
sage: f*g
4*x^4 + 29*x^3 + 18*x^2 + 23*x
 + 6
sage: f*g == f*2+f*(7*x)+f*x^2
True
sage:
```

```
sage: #
sage: #
sage: de
....:
....:
sage:
```

```
Z[]
s a class
ts are polys
h int coeffs
1,4])


7,1])


uilt-in add
```

6

```
sage: f*x   # built-in mul
4*x^3 + x^2 + 3*x
sage: f*x^2
4*x^4 + x^3 + 3*x^2
sage: f*2
8*x^2 + 2*x + 6
sage: f*(7*x)
28*x^3 + 7*x^2 + 21*x
sage: f*g
4*x^4 + 29*x^3 + 18*x^2 + 23*x
 + 6
sage: f*g == f*2+f*(7*x)+f*x^2
True
sage:
```

7

```
sage: # replace
sage: # x^(n+1)
sage: def convol
....:   return (
....:
sage:
```

```
sage: f*x   # built-in mul
4*x^3 + x^2 + 3*x
sage: f*x^2
4*x^4 + x^3 + 3*x^2
sage: f*2
8*x^2 + 2*x + 6
sage: f*(7*x)
28*x^3 + 7*x^2 + 21*x
sage: f*g
4*x^4 + 29*x^3 + 18*x^2 + 23*x
 + 6
sage: f*g == f*2+f*(7*x)+f*x^2
True
sage:
```



```
sage: # replace x^n with
sage: # x^(n+1) with x, e
sage: def convolution(f,g
....:     return (f*g) % (x
....:
sage:
```

```
sage: f*x   # built-in mul
4*x^3 + x^2 + 3*x
sage: f*x^2
4*x^4 + x^3 + 3*x^2
sage: f*2
8*x^2 + 2*x + 6
sage: f*(7*x)
28*x^3 + 7*x^2 + 21*x
sage: f*g
4*x^4 + 29*x^3 + 18*x^2 + 23*x
 + 6
sage: f*g == f*2+f*(7*x)+f*x^2
True
sage:
```



```
sage: # replace x^n with 1,
sage: # x^(n+1) with x, etc.
sage: def convolution(f,g):
....:     return (f*g) % (x^n-1)
....:
sage:
```

```
sage: f*x   # built-in mul
4*x^3 + x^2 + 3*x
sage: f*x^2
4*x^4 + x^3 + 3*x^2
sage: f*2
8*x^2 + 2*x + 6
sage: f*(7*x)
28*x^3 + 7*x^2 + 21*x
sage: f*g
sage: f*g
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sage:
```



```
sage: # replace x^n with 1,
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sage: def convolution(f,g):
....:     return (f*g) % (x^n-1)
....:
sage: n = 3  # global variable
sage:
```

```
sage: f*x   # built-in mul
4*x^3 + x^2 + 3*x
sage: f*x^2
4*x^4 + x^3 + 3*x^2
sage: f*2
8*x^2 + 2*x + 6
sage: f*(7*x)
28*x^3 + 7*x^2 + 21*x
sage: f*g
4*x^4 + 29*x^3 + 18*x^2 + 23*x
 + 6
sage: f*g == f*2+f*(7*x)+f*x^2
True
sage:
```



```
sage: # replace x^n with 1,
sage: # x^(n+1) with x, etc.
sage: def convolution(f,g):
....:     return (f*g) % (x^n-1)
....:
sage: n = 3  # global variable
sage: convolution(f,x)
x^2 + 3*x + 4
sage:
```

```
sage: f*x   # built-in mul
4*x^3 + x^2 + 3*x
sage: f*x^2
4*x^4 + x^3 + 3*x^2
sage: f*2
8*x^2 + 2*x + 6
sage: f*(7*x)
28*x^3 + 7*x^2 + 21*x
sage: f*g
4*x^4 + 29*x^3 + 18*x^2 + 23*x
 + 6
sage: f*g == f*2+f*(7*x)+f*x^2
True
sage:
```



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sage: # replace x^n with 1,
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....:     return (f*g) % (x^n-1)
....:
sage: n = 3  # global variable
sage: convolution(f,x)
x^2 + 3*x + 4
sage: convolution(f,x^2)
3*x^2 + 4*x + 1
sage:
```

```
sage: f*x   # built-in mul
4*x^3 + x^2 + 3*x
sage: f*x^2
4*x^4 + x^3 + 3*x^2
sage: f*2
8*x^2 + 2*x + 6
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28*x^3 + 7*x^2 + 21*x
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4*x^4 + 29*x^3 + 18*x^2 + 23*x
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True
sage:
```



```
sage: # replace x^n with 1,
sage: # x^(n+1) with x, etc.
sage: def convolution(f,g):
....:     return (f*g) % (x^n-1)
....:
sage: n = 3  # global variable
sage: convolution(f,x)
x^2 + 3*x + 4
sage: convolution(f,x^2)
3*x^2 + 4*x + 1
sage: convolution(f,g)
18*x^2 + 27*x + 35
sage:
```

```
*x   # built-in mul
 x^2 + 3*x
*x^2
 x^3 + 3*x^2
*2
 2*x + 6
*(7*x)
+ 7*x^2 + 21*x
*g
 29*x^3 + 18*x^2 + 23*x


*g == f*2+f*(7*x)+f*x^2
```



```
sage: # replace x^n with 1,
sage: # x^(n+1) with x, etc.
sage: def convolution(f,g):
....:     return (f*g) % (x^n-1)
....:
sage: n = 3  # global variable
sage: convolution(f,x)
x^2 + 3*x + 4
sage: convolution(f,x^2)
3*x^2 + 4*x + 1
sage: convolution(f,g)
18*x^2 + 27*x + 35
sage:
```

```
sage: d
....:
....:
....:
....:
sage:
```

```
ilt-in mul
x


x^2


 21*x


 18*x^2 + 23*x


+f*(7*x)+f*x^2
```



```
sage: # replace x^n with 1,
sage: # x^(n+1) with x, etc.
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....:     return (f*g) % (x^n-1)
....:
sage: n = 3  # global variable
sage: convolution(f,x)
x^2 + 3*x + 4
sage: convolution(f,x^2)
3*x^2 + 4*x + 1
sage: convolution(f,g)
18*x^2 + 27*x + 35
sage:
```

```
sage: def random
....:     f = list
....:     for j
....:     return Z
....:
sage:
```

```
l
```

7

```
sage: # replace x^n with 1,
sage: # x^(n+1) with x, etc.
sage: def convolution(f,g):
....:     return (f*g) % (x^n-1)
....:
sage: n = 3  # global variable
sage: convolution(f,x)
x^2 + 3*x + 4
sage: convolution(f,x^2)
3*x^2 + 4*x + 1
sage: convolution(f,g)
18*x^2 + 27*x + 35
sage:
```

```
 23*x
f*x^2
```

8

```
sage: def randompoly():
....:     f = list(randrang
....:     for j in range(
....:     return Zx(f)
....:
sage:
```

```
sage: # replace x^n with 1,
sage: # x^(n+1) with x, etc.
sage: def convolution(f,g):
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....:
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x^2 + 3*x + 4
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....:
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sage: convolution(f,x)
x^2 + 3*x + 4
sage: convolution(f,x^2)
3*x^2 + 4*x + 1
sage: convolution(f,g)
18*x^2 + 27*x + 35
sage:
```



```
sage: def randompoly():
....:     f = list(randrange(3)-1
....:       for j in range(n))
....:     return Zx(f)
....:
sage: n = 7
sage:
```

```
sage: # replace x^n with 1,
sage: # x^(n+1) with x, etc.
sage: def convolution(f,g):
....:     return (f*g) % (x^n-1)
....:
sage: n = 3  # global variable
sage: convolution(f,x)
x^2 + 3*x + 4
sage: convolution(f,x^2)
3*x^2 + 4*x + 1
sage: convolution(f,g)
18*x^2 + 27*x + 35
sage:
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....:       for j in range(n))
....:     return Zx(f)
....:
sage: n = 7
sage: randompoly()
-x^3 - x^2 - x - 1
sage:
```

```
sage: # replace x^n with 1,
sage: # x^(n+1) with x, etc.
sage: def convolution(f,g):
....:     return (f*g) % (x^n-1)
....:
sage: n = 3  # global variable
sage: convolution(f,x)
x^2 + 3*x + 4
sage: convolution(f,x^2)
3*x^2 + 4*x + 1
sage: convolution(f,g)
18*x^2 + 27*x + 35
sage:
```



```
sage: def randompoly():
....:     f = list(randrange(3)-1
....:       for j in range(n))
....:     return Zx(f)
....:
sage: n = 7
sage: randompoly()
-x^3 - x^2 - x - 1
sage: randompoly()
x^6 + x^5 + x^3 - x
sage:
```

```
sage: # replace x^n with 1,
sage: # x^(n+1) with x, etc.
sage: def convolution(f,g):
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....:
sage: n = 7
sage: randompoly()
-x^3 - x^2 - x - 1
sage: randompoly()
x^6 + x^5 + x^3 - x
sage: randompoly()
-x^6 + x^5 + x^4 - x^3 - x^2 +
 x + 1
sage:
```

```
 replace x^n with 1,
 x^(n+1) with x, etc.
ef convolution(f,g):
 return (f*g) % (x^n-1)


 = 3  # global variable
onvolution(f,x)
*x + 4
onvolution(f,x^2)
 4*x + 1
onvolution(f,g)
+ 27*x + 35
```



```
sage: def randompoly():
....:     f = list(randrange(3)-1
....:       for j in range(n))
....:     return Zx(f)
....:
sage: n = 7
sage: randompoly()
-x^3 - x^2 - x - 1
sage: randompoly()
x^6 + x^5 + x^3 - x
sage: randompoly()
-x^6 + x^5 + x^4 - x^3 - x^2 +
 x + 1
sage:
```

Will use

Some ch
in submi

$n = 701$

$n = 743$

$n = 761$

```
x^n with 1,
with x, etc.
ution(f,g):
f*g) % (x^n-1)


lobal variable
n(f,x)


n(f,x^2)


n(f,g)
35
```



```
sage: def randompoly():
....:     f = list(randrange(3)-1
....:       for j in range(n))
....:     return Zx(f)
....:
sage: n = 7
sage: randompoly()
-x^3 - x^2 - x - 1
sage: randompoly()
x^6 + x^5 + x^3 - x
sage: randompoly()
-x^6 + x^5 + x^4 - x^3 - x^2 +
 x + 1
sage:
```

Will use bigger n f

Some choices of n
in submissions to

$n = 701$ for NTRU

$n = 743$ for NTRU

$n = 761$ for sntru

```
1,
etc.
):
^n-1)


riable
```

8

```
sage: def randompoly():
....:     f = list(randrange(3)-1
....:       for j in range(n))
....:     return Zx(f)
....:
sage: n = 7
sage: randompoly()
-x^3 - x^2 - x - 1
sage: randompoly()
x^6 + x^5 + x^3 - x
sage: randompoly()
-x^6 + x^5 + x^4 - x^3 - x^2 +
 x + 1
sage:
```

9

Will use bigger n for security

Some choices of n
in submissions to NIST:

$n = 701$ for NTRU HRSS.

$n = 743$ for NTRUEncrypt.

$n = 761$ for `sntrup4591761`

```
sage: def randompoly():
....:     f = list(randrange(3)-1
....:       for j in range(n))
....:     return Zx(f)
....:
sage: n = 7
sage: randompoly()
-x^3 - x^2 - x - 1
sage: randompoly()
sage: randompoly()
x^6 + x^5 + x^3 - x
sage: randompoly()
-x^6 + x^5 + x^4 - x^3 - x^2 +
 x + 1
sage:
```


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```
sage: def randompoly():
....:     f = list(randrange(3)-1
....:       for j in range(n))
....:     return Zx(f)
....:
sage: n = 7
sage: randompoly()
-x^3 - x^2 - x - 1
sage: randompoly()
x^6 + x^5 + x^3 - x
sage: randompoly()
-x^6 + x^5 + x^4 - x^3 - x^2 +
 x + 1
sage:
```


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Overkill against attack algorithms known today, even for future attacker with quantum computer.

```
sage: def randompoly():
....:     f = list(randrange(3)-1
....:       for j in range(n))
....:     return Zx(f)
....:
sage: n = 7
sage: randompoly()
-x^3 - x^2 - x - 1
sage: randompoly()
x^6 + x^5 + x^3 - x
sage: randompoly()
-x^6 + x^5 + x^4 - x^3 - x^2 +
 x + 1
sage:
```


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Can we find better algorithms?

```
sage: def randompoly():
....:     f = list(randrange(3)-1
....:       for j in range(n))
....:     return Zx(f)
....:
sage: n = 7
sage: randompoly()
-x^3 - x^2 - x - 1
sage: randompoly()
x^6 + x^5 + x^3 - x
sage: randompoly()
-x^6 + x^5 + x^4 - x^3 - x^2 +
 x + 1
sage:
```


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Overkill against attack algorithms known today, even for future attacker with quantum computer.

Can we find better algorithms?

1998 NTRU paper took $n = 503$.

```
ef randompoly():
 f = list(randrange(3)-1
   for j in range(n))
 return Zx(f)


 = 7
andompoly()
x^2 - x - 1
andompoly()
^5 + x^3 - x
andompoly()
x^5 + x^4 - x^3 - x^2 +
```


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Can we find better algorithms?

1998 NTRU paper took $n = 503$.

Modular

For integ
Sage's "
outputs

Matches

```
poly():
(randrange(3)-1
in range(n))
x(f)



()
 1
()
- x
()
 - x^3 - x^2 +
```


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Can we find better algorithms?

1998 NTRU paper took $n = 503$.

Modular reduction

For integers u, q w
Sage's "u%q" alwa
outputs between 0

Matches standard

```
ge(3)-1
(n))
```

```
x^2 +
```


Will use bigger n for security.

Some choices of n
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attacker with quantum computer.

Can we find better algorithms?

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Modular reduction

For integers u, q with q > 0
Sage's "u%q" always produc
outputs between 0 and q −

Matches standard math defi

Will use bigger n for security.

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Overkill against attack algorithms
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attacker with quantum computer.

Can we find better algorithms?

1998 NTRU paper took $n = 503$.

Modular reduction

For integers $\mathtt{u}$, $\mathtt{q}$ with $\mathtt{q} > 0$,
Sage's "u%q" always produces
outputs between 0 and $\mathtt{q} - 1$.

Matches standard math definition.

Will use bigger n for security.

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in submissions to NIST:

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Can we find better algorithms?

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Modular reduction

For integers u, q with $q > 0$,
Sage's "u%q" always produces
outputs between 0 and $q - 1$.

Matches standard math definition.

Warning: Typically
$u < 0$ produces $u\%q < 0$
in lower-level languages, so
nonzero output leaks input sign.

Will use bigger n for security.

Some choices of n
in submissions to NIST:

$n = 701$ for NTRU HRSS.

$n = 743$ for NTRUEncrypt.

$n = 761$ for `sntrup4591761`.

Overkill against attack algorithms
known today, even for future
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Can we find better algorithms?

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Warning: For polynomials u,
Sage can make the same mistake.

bigger n for security.

oices of n
issions to NIST:

for NTRU HRSS.
for NTRUEncrypt.
for `sntrup4591761`.

against attack algorithms
oday, even for future
with quantum computer.

find better algorithms?

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Sage's "u%q" always produces
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Warning: For polynomials u,
Sage can make the same mistake.

```
sage: d
sage:
sage:
sage:
sage:
sage:
```

for security.

NIST:

U HRSS.

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ntum computer.

r algorithms?

r took $n = 503$.

Modular reduction

For integers u, q with $q > 0$,
Sage's "u%q" always produces
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$u < 0$ produces $u\%q < 0$
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nonzero output leaks input sign.

Warning: For polynomials u,
Sage can make the same mistake.

```
sage: def balanc
sage:     g=list((
sage:     -q//2 fo
sage:     return Z
sage:
sage:
```

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ithms

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ns?

= 503.

Modular reduction

For integers u, q with $q > 0$,
Sage's "u%q" always produces
outputs between 0 and $q - 1$.

Matches standard math definition.

Warning: Typically
$u < 0$ produces $u\%q < 0$
in lower-level languages, so
nonzero output leaks input sign.

Warning: For polynomials u,
Sage can make the same mistake.


```
sage: def balancedmod(f,q
sage:     g=list(((f[i]+q//
sage:     -q//2 for i in ra
sage:     return Zx(g)
sage:
sage:
```

Modular reduction

For integers u, q with $q > 0$,
Sage's "u%q" always produces
outputs between 0 and $q - 1$.

Matches standard math definition.

Warning: Typically
$u < 0$ produces $u\%q < 0$
in lower-level languages, so
nonzero output leaks input sign.

Warning: For polynomials u,
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sage: def balancedmod(f,q):
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sage: def invert
....:     Fp = Int
....:     Fpx = Zx
....:     T = Fpx.
....:     return Z
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sage:
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12

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```

```
def inve
  assert
  g = in
  M = ba
  C = co
  while
    r =
    if
      g =
```

Exercise
invertm
Hint: Co

```
edmod(f,q):
(f[i]+q//2)%q)
r i in range(n))
x(g)


9*x



% 200


d(u,200)
```



```
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```
def invertmodpow
    assert q.is_po
    g = invertmodp
    M = balancedmo
    C = convolutio
    while True:
        r = M(C(g,f)
        if r == 1: r
        g = M(C(g,2-
```

Exercise: Figure o
`invertmodpowerc`
Hint: Compare `r`

```
):
2)%q)
nge(n))
```



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```
def invertmodpowerof2(f,q
    assert q.is_power_of(2)
    g = invertmodprime(f,2)
    M = balancedmod
    C = convolution
    while True:
        r = M(C(g,f),q)
        if r == 1: return g
        g = M(C(g,2-r),q)
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Exercise: Figure out how `invertmodpowerof2` works.

Hint: Compare `r` to previous

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sage:
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modprime(f,p):
egers(p)
.change_ring(Fp)
quotient(x^n-1)
x(lift(1/T(f)))


poly()
tmodprime(f,3)
n(f,f3)
3*x^4 + 3*x^3 +
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```

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r),q)
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NTRU key generat

Parameters:

n, positive integer

q, power of 2 (e.g

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1
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NTRU key generation

Parameters:

n, positive integer (e.g., 701

q, power of 2 (e.g., 4096).

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1
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Parameters:

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```
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sage:
```


NTRU key generation

Parameters:

n, positive integer (e.g., 701);

q, power of 2 (e.g., 4096).

Secret key:

random n-coeff polynomial a;

random n-coeff polynomial d;

all coefficients in $\{-1, 0, 1\}$.

```
sage: n = 7
sage: q = 256
sage: f = randompoly()
sage: f
-x^6 - x^4 + x^2 + x - 1
sage: g = invertmodpowerof2(f,q)
sage: g
47*x^6 + 126*x^5 - 54*x^4 -
 87*x^3 - 36*x^2 - 58*x + 61
sage: convolution(f,g)
-256*x^5 - 256*x^4 + 256*x + 257
sage: balancedmod(_,q)
1
sage:
```


NTRU key generation

Parameters:

n, positive integer (e.g., 701);

q, power of 2 (e.g., 4096).

Secret key:

random n-coeff polynomial a;

random n-coeff polynomial d;

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Require d invertible mod q.

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 = 256
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 = invertmodpowerof2(f,q)


+ 126*x^5 - 54*x^4 -
 - 36*x^2 - 58*x + 61
onvolution(f,g)
5 - 256*x^4 + 256*x + 257
alancedmod(_,q)
```


NTRU key generation

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```
def keyp
  while
    try
      d
      d3
      dq
      b
    exc
      pa
a = ra
publi


secre
return
```

```
poly()

 + x - 1

modpowerof2(f,q)

 - 54*x^4 -

 - 58*x + 61

n(f,g)

^4 + 256*x + 257

d(_,q)
```


NTRU key generation

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```
def keypair():
  while True:
    try:
      d = random
      d3 = inver
      dq = inver
      break
    except:
      pass
  a = randompoly
  publickey = ba
                con
  secretkey = d,
  return publick
```

```
f2(f,q)

 -
 61

x + 257
```

NTRU key generation

Parameters:

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```
def keypair():
  while True:
    try:
      d = randompoly()
      d3 = invertmodprime
      dq = invertmodpower
      break
    except:
      pass
  a = randompoly()
  publickey = balancedmod
              convolution(
  secretkey = d,d3
  return publickey,secret
```

NTRU key generation

Parameters:

n, positive integer (e.g., 701);

q, power of 2 (e.g., 4096).

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      break
    except:
      pass
  a = randompoly()
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                convolution(a,dq),q)
  secretkey = d,d3
  return publickey,secretkey
```

key generation

ters:

ve integer (e.g., 701);

r of 2 (e.g., 4096).

ey:

n-coeff polynomial a;

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icients in $\{-1, 0, 1\}$.

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```

```
sage: A
sage:
```

tion

(e.g., 701);
., 4096).

olynomial a;
olynomial d;
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le mod q.
le mod 3.

$3a/d$ in the ring
$^n - 1$).

16

```
def keypair():
  while True:
    try:
      d = randompoly()
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      break
    except:
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  publickey = balancedmod(3 *
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  secretkey = d,d3
  return publickey,secretkey
```

17

```
sage: A,secretke
sage:
```

16

```
def keypair():
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      break
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      pass
  a = randompoly()
  publickey = balancedmod(3 *
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  secretkey = d,d3
  return publickey,secretkey
```

17

```
sage: A,secretkey = keypa
sage:
```

```
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  return publickey,secretkey
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```
sage: A,secretkey = keypair()
sage:
```

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```



```
sage: A,secretkey = keypair()
sage: A
-126*x^6 - 31*x^5 - 118*x^4 -
 33*x^3 + 73*x^2 - 16*x + 7
sage:
```

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sage: d
-x^6 + x^5 - x^4 + x^3 - 1
sage:
```

```
def keypair():
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    try:
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 33*x^3 + 73*x^2 - 16*x + 7
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sage: d
-x^6 + x^5 - x^4 + x^3 - 1
sage: convolution(d,A)
-3*x^6 + 253*x^5 + 253*x^3 -
 253*x^2 - 3*x - 3
sage:
```

```
def keypair():
  while True:
    try:
      d = randompoly()
      d3 = invertmodprime(d,3)
      dq = invertmodpowerof2(d,q)
      break
    except:
      pass
  a = randompoly()
  publickey = balancedmod(3 *
              convolution(a,dq),q)
  secretkey = d,d3
  return publickey,secretkey
```



```
sage: A,secretkey = keypair()
sage: A
-126*x^6 - 31*x^5 - 118*x^4 -
 33*x^3 + 73*x^2 - 16*x + 7
sage: d,d3 = secretkey
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-x^6 + x^5 - x^4 + x^3 - 1
sage: convolution(d,A)
-3*x^6 + 253*x^5 + 253*x^3 -
 253*x^2 - 3*x - 3
sage: balancedmod(_,q)
-3*x^6 - 3*x^5 - 3*x^3 + 3*x^2
 - 3*x - 3
sage:
```

```
pair():
 True:
:
 = randompoly()
3 = invertmodprime(d,3)
q = invertmodpowerof2(d,q)
reak
ept:
ass
andompoly()
ckey = balancedmod(3 *
       convolution(a,dq),q)
tkey = d,d3
n publickey,secretkey
```



```
sage: A,secretkey = keypair()
sage: A
-126*x^6 - 31*x^5 - 118*x^4 -
 33*x^3 + 73*x^2 - 16*x + 7
sage: d,d3 = secretkey
sage: d
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sage: balancedmod(_,q)
-3*x^6 - 3*x^5 - 3*x^3 + 3*x^2
 - 3*x - 3
sage:
```

NTRU e

One mo

w, posit

```
poly()
tmodprime(d,3)
tmodpowerof2(d,q)
()
lancedmod(3 *
volution(a,dq),q)
d3
ey,secretkey
```



```
sage: A,secretkey = keypair()
sage: A
-126*x^6 - 31*x^5 - 118*x^4 -
 33*x^3 + 73*x^2 - 16*x + 7
sage: d,d3 = secretkey
sage: d
-x^6 + x^5 - x^4 + x^3 - 1
sage: convolution(d,A)
-3*x^6 + 253*x^5 + 253*x^3 -
 253*x^2 - 3*x - 3
sage: balancedmod(_,q)
-3*x^6 - 3*x^5 - 3*x^3 + 3*x^2
 - 3*x - 3
sage:
```

NTRU encryption

One more paramet

w, positive integer

```
e(d,3)
rof2(d,q)
(3 *
(a,dq),q)
key
```

```
sage: A,secretkey = keypair()
sage: A
-126*x^6 - 31*x^5 - 118*x^4 -
 33*x^3 + 73*x^2 - 16*x + 7
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sage: d
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sage: convolution(d,A)
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 253*x^2 - 3*x - 3
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-3*x^6 - 3*x^5 - 3*x^3 + 3*x^2
 - 3*x - 3
sage:
```


NTRU encryption

One more parameter:

w, positive integer (e.g., 46

```
sage: A,secretkey = keypair()
sage: A
-126*x^6 - 31*x^5 - 118*x^4 -
 33*x^3 + 73*x^2 - 16*x + 7
sage: d,d3 = secretkey
sage: d
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 253*x^2 - 3*x - 3
sage: balancedmod(_,q)
-3*x^6 - 3*x^5 - 3*x^3 + 3*x^2
 - 3*x - 3
sage:
```


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 253*x^2 - 3*x - 3
sage: balancedmod(_,q)
-3*x^6 - 3*x^5 - 3*x^3 + 3*x^2
 - 3*x - 3
sage:
```


NTRU encryption

One more parameter:
w, positive integer (e.g., 467).

Message for encryption:
n-coeff weight-w polynomial c
with all coeffs in $\{-1, 0, 1\}$.

"Weight w": w nonzero coeffs,
$n - w$ zero coeffs.

```
sage: A,secretkey = keypair()
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-126*x^6 - 31*x^5 - 118*x^4 -
 33*x^3 + 73*x^2 - 16*x + 7
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sage: d
-x^6 + x^5 - x^4 + x^3 - 1
sage: convolution(d,A)
-3*x^6 + 253*x^5 + 253*x^3 -
 253*x^2 - 3*x - 3
sage: balancedmod(_,q)
-3*x^6 - 3*x^5 - 3*x^3 + 3*x^2
 - 3*x - 3
sage:
```


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Ciphertext: $C = Ab + c$ in R_q
where b is chosen randomly
from the set of messages.

```
,secretkey = keypair()


6 - 31*x^5 - 118*x^4 -
 + 73*x^2 - 16*x + 7
,d3 = secretkey


x^5 - x^4 + x^3 - 1
onvolution(d,A)
+ 253*x^5 + 253*x^3 -
2 - 3*x - 3
alancedmod(_,q)
- 3*x^5 - 3*x^3 + 3*x^2
- 3
```


NTRU encryption

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```
sage: de
....:
....:
....:
....:
....:
....:
....:
....:
....:
....:
sage: w
sage: ra
-x^6 - 1
sage:
```

```
y = keypair()


5 - 118*x^4 -
 - 16*x + 7
retkey


 + x^3 - 1
n(d,A)
 + 253*x^3 -
 3
d(_,q)
 3*x^3 + 3*x^2
```


NTRU encryption

One more parameter:
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Message for encryption:
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Ciphertext: $C = Ab + c$ in R_q
where b is chosen randomly
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```
sage: def random
....:     R = rand
....:     assert w
....:     c = n*[0
....:     for j in
....:         while
....:             r =
....:             if n
....:                 c[r] =
....:     return Z
....:
sage: w = 5
sage: randommess
-x^6 - x^5 + x^4
sage:
```

```
ir()


^4 -
 7



1


3 -


3*x^2
```

NTRU encryption

One more parameter:
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from the set of messages.


```
sage: def randommessage()
....:     R = randrange
....:     assert w <= n
....:     c = n*[0]
....:     for j in range(w)
....:         while True:
....:             r = R(n)
....:             if not c[r]:
....:                 c[r] = 1-2*R(2)
....:     return Zx(c)
....:
sage: w = 5
sage: randommessage()
-x^6 - x^5 + x^4 + x^3 -
sage:
```

NTRU encryption

One more parameter:
w, positive integer (e.g., 467).

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....:             r = R(n)
....:             if not c[r]: break
....:         c[r] = 1-2*R(2)
....:     return Zx(c)
....:
sage: w = 5
sage: randommessage()
-x^6 - x^5 + x^4 + x^3 - x^2
sage:
```

encryption

re parameter:
ive integer (e.g., 467).

e for encryption:
weight-w polynomial c
coeffs in $\{-1, 0, 1\}$.

w": w nonzero coeffs,
ero coeffs.

ext: $C = Ab + c$ in R_q
is chosen randomly
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....:
sage: w = 5
sage: randommessage()
-x^6 - x^5 + x^4 + x^3 - x^2
sage:
```

```
sage: de
....:
....:
....:
....:
....:
sage:
```

ter:

r (e.g., 467).

ption:

polynomial c

$\{-1, 0, 1\}$.

onzero coeffs,

$Ab + c$ in R_q

randomly

essages.

19

```
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....:     return Zx(c)
....:
sage: w = 5
sage: randommessage()
-x^6 - x^5 + x^4 + x^3 - x^2
sage:
```

20

```
sage: def encryp
....:     b = rand
....:     Ab = con
....:     C = bala
....:     return C
....:
sage:
```

7).

l c

effs,

R_q

19

```
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sage: w = 5
sage: randommessage()
-x^6 - x^5 + x^4 + x^3 - x^2
sage:
```

20

```
sage: def encrypt(c,A):
....:     b = randommessage
....:     Ab = convolution(
....:     C = balancedmod(A
....:     return C
....:
sage:
```

```
sage: def randommessage():
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....:             if not c[r]: break
....:         c[r] = 1-2*R(2)
....:     return Zx(c)
....:
sage: w = 5
sage: randommessage()
-x^6 - x^5 + x^4 + x^3 - x^2
sage:
```



```
sage: def encrypt(c,A):
....:     b = randommessage()
....:     Ab = convolution(A,b)
....:     C = balancedmod(Ab + c,q)
....:     return C
....:
sage:
```

```
sage: def randommessage():
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....:     assert w <= n
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....:         while True:
....:             r = R(n)
....:             if not c[r]: break
....:         c[r] = 1-2*R(2)
....:     return Zx(c)
....:
....:
sage: w = 5
sage: randommessage()
-x^6 - x^5 + x^4 + x^3 - x^2
sage:
```



```
sage: def encrypt(c,A):
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....:     Ab = convolution(A,b)
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....:     return C
....:
sage: A,secretkey = keypair()
sage:
```

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....:
....:
sage: w = 5
sage: randommessage()
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sage:
```



```
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....:     return C
....:
sage: A,secretkey = keypair()
sage: c = randommessage()
sage:
```

```
sage: def randommessage():
....:     R = randrange
....:     assert w <= n
....:     c = n*[0]
....:     for j in range(w):
....:         while True:
....:             r = R(n)
....:             if not c[r]: break
....:         c[r] = 1-2*R(2)
....:     return Zx(c)
....:
....:
sage: w = 5
sage: randommessage()
-x^6 - x^5 + x^4 + x^3 - x^2
sage:
```



```
sage: def encrypt(c,A):
....:     b = randommessage()
....:     Ab = convolution(A,b)
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sage: A,secretkey = keypair()
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 76*x^3 - 77*x^2 + 15*x - 113
sage:
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NTRU d

Comput

```
message():
range
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]
 range(w):
True:
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NTRU decryption

Compute $dC = 3a$

```
:
:
break
x^2
```



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 return C


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 = encrypt(c,A)


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```
sage: de
....:
....:
....:
....:
....:
....:
sage:
```

```
t(c,A):
ommessage()
volution(A,b)
ncedmod(Ab + c,q)


y = keypair()
message()
t(c,A)


+ 31*x^4 -
 + 15*x - 113
```


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```
sage: def decryp
....:     M = ba
....:     f,r =
....:     u=M(co
....:     c=M(co
....:     return
....:
sage:
```

```
e()
(A,b)
b + c,q)


ir()


-
 113
```


NTRU decryption

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```
sage: def decrypt(C,secre
....:     M = balancedmod
....:     f,r = secretkey
....:     u=M(convolution
....:     c=M(convolution
....:     return c
....:
sage:
```

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sage: def decrypt(C,secretkey):
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c in $R = \mathbf{Z}[x]/(x^n - 1)$.

modulo 3: dc in R_3.

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```
sage: n
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sage: q
sage:
```

$ab + dc$ in R_q.

ll coeffs,

t very big.

ffs of $3ab + dc$

and $q/2 - 1$.

R_q reveals

$\mathbf{Z}[x]/(x^n - 1)$.

dc in R_3.

n R_3

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```

```
sage: n = 7
sage: w = 5
sage: q = 256
sage:
```

R_q.

$+ dc$

$- 1$.

s

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.

1,

22

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23

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 83*x^3 + 40*x^2 + 108*x - 54
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x^5 + x^4 - x^3 + x + 1
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sage:
sage:
```

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sage: a3
3*x^2 - 3*x
```

```
ef decrypt(C,secretkey):
    M = balancedmod
    f,r = secretkey
    u=M(convolution(C,f),q)
    c=M(convolution(u,r),3)
    return c


^4 - x^3 + x + 1
ecrypt(C,secretkey)
^4 - x^3 + x + 1
```



```
sage: n = 7
sage: w = 5
sage: q = 256
sage: A,secretkey = keypair()
sage: A
-101*x^6 - 76*x^5 - 90*x^4 -
 83*x^3 + 40*x^2 + 108*x - 54
sage: d,d3 = secretkey
sage: d
x^5 + x^4 - x^3 + x - 1
sage: conv = convolution
sage: M = balancedmod
sage: a3 = M(conv(d,A),q)
sage: a3
3*x^2 - 3*x
```

```
sage: c
sage:
```

```
t(C,secretkey):
lancedmod
secretkey
nvolution(C,f),q)
nvolution(u,r),3)
 c


+ x + 1
secretkey)
+ x + 1
```



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```

```
sage: c = random
sage:
```

```
etkey):
1
r
1(C,f),q)
1(u,r),3)
)
```

```
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sage: a3
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```



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sage: b = randommessage()
sage: C = M(conv(A,b)+c,q)
sage:
```

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 72*x^3 - 37*x^2 + 16*x + 119
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```

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x^5 + x^4 - x^3 + x - 1
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 72*x^3 - 37*x^2 + 16*x + 119
sage: u = M(conv(C,d),q)
sage: u
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
sage:
```

```
sage: n = 7
sage: w = 5
sage: q = 256
sage: A,secretkey = keypair()
sage: A
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 83*x^3 + 40*x^2 + 108*x - 54
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sage: d
x^5 + x^4 - x^3 + x - 1
sage: conv = convolution
sage: M = balancedmod
sage: a3 = M(conv(d,A),q)
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3*x^2 - 3*x
```



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sage: u = M(conv(C,d),q)
sage: u
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
sage: conv(a3,b)+conv(c,d)
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
```

```
 = 7
 = 5
 = 256
,secretkey = keypair()

6 - 76*x^5 - 90*x^4 -
 + 40*x^2 + 108*x - 54
,d3 = secretkey

^4 - x^3 + x - 1
onv = convolution
 = balancedmod
3 = M(conv(d,A),q)
3
 3*x
```



```
sage: c = randommessage()
sage: b = randommessage()
sage: C = M(conv(A,b)+c,q)
sage: C
-57*x^6 + 28*x^5 + 114*x^4 +
 72*x^3 - 37*x^2 + 16*x + 119
sage: u = M(conv(C,d),q)
sage: u
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
sage: conv(a3,b)+conv(c,d)
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
```

```
sage: M
x^6 - x
 + 1
sage:
```

```
y = keypair()


5 - 90*x^4 -
 + 108*x - 54
retkey


+ x - 1
volution
edmod
v(d,A),q)
```



```
sage: c = randommessage()
sage: b = randommessage()
sage: C = M(conv(A,b)+c,q)
sage: C
-57*x^6 + 28*x^5 + 114*x^4 +
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-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
sage: conv(a3,b)+conv(c,d)
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
```

```
sage: M(u,3)
x^6 - x^5 + x^4
 + 1
sage:
```

```
ir()

4 -

- 54
```

```
sage: c = randommessage()
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sage: C = M(conv(A,b)+c,q)
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-57*x^6 + 28*x^5 + 114*x^4 +
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 4*x^2 + 5*x + 1
sage: conv(a3,b)+conv(c,d)
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
```



```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x
 + 1
sage:
```

```
sage: c = randommessage()
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sage: C = M(conv(A,b)+c,q)
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-57*x^6 + 28*x^5 + 114*x^4 +
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 4*x^2 + 5*x + 1
```



```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage:
```

```
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-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
```



```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
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```

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 4*x^2 + 5*x + 1
```



```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: conv(M(u,3),d3)
x^6 - x^5 - x^4 - 3*x^3 - x^2 +
 x - 3
sage:
```

```
sage: c = randommessage()
sage: b = randommessage()
sage: C = M(conv(A,b)+c,q)
sage: C
-57*x^6 + 28*x^5 + 114*x^4 +
 72*x^3 - 37*x^2 + 16*x + 119
sage: u = M(conv(C,d),q)
sage: u
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
sage: conv(a3,b)+conv(c,d)
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
```



```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: conv(M(u,3),d3)
x^6 - x^5 - x^4 - 3*x^3 - x^2 +
 x - 3
sage: M(_,3)
x^6 - x^5 - x^4 - x^2 + x
sage:
```

```
sage: c = randommessage()
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-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
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sage: conv(a3,b)+conv(c,d)
-8*x^6 + 2*x^5 + 4*x^4 - x^3 -
 4*x^2 + 5*x + 1
```



```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: conv(M(u,3),d3)
x^6 - x^5 - x^4 - 3*x^3 - x^2 +
 x - 3
sage: M(_,3)
x^6 - x^5 - x^4 - x^2 + x
sage: c
x^6 - x^5 - x^4 - x^2 + x
sage:
```

```
= randommessage()
= randommessage()
= M(conv(A,b)+c,q)


+ 28*x^5 + 114*x^4 +
- 37*x^2 + 16*x + 119
= M(conv(C,d),q)


+ 2*x^5 + 4*x^4 - x^3 -
+ 5*x + 1
onv(a3,b)+conv(c,d)
+ 2*x^5 + 4*x^4 - x^3 -
+ 5*x + 1
```



```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: conv(M(u,3),d3)
x^6 - x^5 - x^4 - 3*x^3 - x^2 +
 x - 3
sage: M(_,3)
x^6 - x^5 - x^4 - x^2 + x
sage: c
x^6 - x^5 - x^4 - x^2 + x
sage:
```

Does de

All coeff

All coeff

and exac

```
message()
message()
(A,b)+c,q)


 + 114*x^4 +
 + 16*x + 119
(C,d),q)


 4*x^4 - x^3 -


+conv(c,d)
 4*x^4 - x^3 -
```



```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: conv(M(u,3),d3)
x^6 - x^5 - x^4 - 3*x^3 - x^2 +
 x - 3
sage: M(_,3)
x^6 - x^5 - x^4 - x^2 + x
sage: c
x^6 - x^5 - x^4 - x^2 + x
sage:
```

Does decryption a

All coeffs of a are
All coeffs of b are
and exactly w are

```
)

4 +

- 119

x^3 -

)

x^3 -
```



```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: conv(M(u,3),d3)
x^6 - x^5 - x^4 - 3*x^3 - x^2 +
 x - 3
sage: M(_,3)
x^6 - x^5 - x^4 - x^2 + x
sage: c
x^6 - x^5 - x^4 - x^2 + x
sage:
```

Does decryption always work

All coeffs of a are in $\{-1, 0,$

All coeffs of b are in $\{-1, 0,$

and exactly w are nonzero.

```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: conv(M(u,3),d3)
x^6 - x^5 - x^4 - 3*x^3 - x^2 +
 x - 3
sage: M(_,3)
x^6 - x^5 - x^4 - x^2 + x
sage: c
x^6 - x^5 - x^4 - x^2 + x
sage:
```


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All coeffs of a are in $\{-1, 0, 1\}$.
All coeffs of b are in $\{-1, 0, 1\}$,
and exactly w are nonzero.

```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: conv(M(u,3),d3)
x^6 - x^5 - x^4 - 3*x^3 - x^2 +
 x - 3
sage: M(_,3)
x^6 - x^5 - x^4 - x^2 + x
sage: c
x^6 - x^5 - x^4 - x^2 + x
sage:
```


Does decryption always work?

All coeffs of a are in $\{-1, 0, 1\}$.
All coeffs of b are in $\{-1, 0, 1\}$,
and exactly w are nonzero.

Each coeff of ab in R
has absolute value at most w.

```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: conv(M(u,3),d3)
x^6 - x^5 - x^4 - 3*x^3 - x^2 +
 x - 3
sage: M(_,3)
x^6 - x^5 - x^4 - x^2 + x
sage: c
x^6 - x^5 - x^4 - x^2 + x
sage:
```


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Each coeff of ab in R
has absolute value at most w.
(Same argument would work for
b of any weight, a of weight w.)

```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: conv(M(u,3),d3)
x^6 - x^5 - x^4 - 3*x^3 - x^2 +
 x - 3
sage: M(_,3)
x^6 - x^5 - x^4 - x^2 + x
sage: c
x^6 - x^5 - x^4 - x^2 + x
sage:
```


Does decryption always work?

All coeffs of a are in $\{-1, 0, 1\}$.
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(Same argument would work for
b of any weight, a of weight w.)

Similar comments for d, c.
Each coeff of $3ab + dc$ in R
has absolute value at most $4w$.

```
sage: M(u,3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: M(conv(c,d),3)
x^6 - x^5 + x^4 - x^3 - x^2 - x
 + 1
sage: conv(M(u,3),d3)
x^6 - x^5 - x^4 - 3*x^3 - x^2 +
 x - 3
sage: M(_,3)
x^6 - x^5 - x^4 - x^2 + x
sage: c
x^6 - x^5 - x^4 - x^2 + x
sage:
```


Does decryption always work?

All coeffs of a are in $\{-1, 0, 1\}$.
All coeffs of b are in $\{-1, 0, 1\}$,
and exactly w are nonzero.

Each coeff of ab in R
has absolute value at most w.
(Same argument would work for
b of any weight, a of weight w.)

Similar comments for d, c.
Each coeff of $3ab + dc$ in R
has absolute value at most $4w$.

e.g. $w = 467$: at most 1868.
Decryption works for $q = 4096$.

```
(u,3)
^5 + x^4 - x^3 - x^2 - x


(conv(c,d),3)
^5 + x^4 - x^3 - x^2 - x


onv(M(u,3),d3)
^5 - x^4 - 3*x^3 - x^2 +


(_,3)
^5 - x^4 - x^2 + x


^5 - x^4 - x^2 + x
```


Does decryption always work?

All coeffs of a are in $\{-1, 0, 1\}$.
All coeffs of b are in $\{-1, 0, 1\}$,
and exactly w are nonzero.

Each coeff of ab in R
has absolute value at most w.
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Similar comments for d, c.
Each coeff of $3ab + dc$ in R
has absolute value at most $4w$.

e.g. $w = 467$: at most 1868.
Decryption works for $q = 4096$.

What ab

```
- x^3 - x^2 - x


),3)
- x^3 - x^2 - x


),d3)
- 3*x^3 - x^2 +

- x^2 + x

- x^2 + x
```


Does decryption always work?

All coeffs of a are in $\{-1, 0, 1\}$.
All coeffs of b are in $\{-1, 0, 1\}$,
and exactly w are nonzero.

Each coeff of ab in R
has absolute value at most w.
(Same argument would work for
b of any weight, a of weight w.)

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e.g. $w = 467$: at most 1868.
Decryption works for $q = 4096$.

What about $w =$

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Does decryption always work?

All coeffs of a are in $\{-1, 0, 1\}$.
All coeffs of b are in $\{-1, 0, 1\}$,
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Attacker changes c to
$c \pm 1$, $c \pm x$, ..., $c \pm x^{n-1}$;
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$c \pm 3$, etc.

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This changes $3ab + dc$: adds
$\pm d$, $\pm xd$, …, $\pm x^{n-1} d$;
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KCL: $\approx 2^{-60}$.

Ding: $\approx 2^{-60}$, only IND-CPA

Current debates about what decryption failure prob is small enough; whether decryption failure probabiliti were calculated correctly; et

How to handle decryption failures

Eliminating invalid messages is not enough: remember attack using decryption failures for random valid messages.

NIST encryption submissions vary in failure rates.

NTRU HRSS, NTRU Prime, Odd Manhattan choose q to eliminate decryption failures.

LIMA tried to eliminate decryption failures, but failed.

More claimed failure rates:

LOTUS: $<2^{-256}$.

New Hope submission: $<2^{-213}$.

KINDI: 2^{-165}.

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Choose random message.
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Enumerate all $H(-(A/3)d_2)$.
Enumerate all $H((A/3)d_1)$.
Search for collisions.
Only about $3^{n/2}$ computations;
but beware cost of memory.

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Enumerate all $H(-(A/3)d_2)$.
Enumerate all $H((A/3)d_1)$.
Search for collisions.
Only about $3^{n/2}$ computations;
but beware cost of memory.

Lattices

s equivalent to
),
^2d), etc.

$3^n/n$ choices.

:
$^n_w)2^w \approx 2^{1106.09}$;
$3^n \approx 2^{1111.06}$;
$3^n/n \approx 2^{1101.61}$.

re equivalences!

smaller then
be faster.

Collision attacks

Write d as $d_1 + d_2$ where
$d_1 =$ bottom $\lceil n/2 \rceil$ terms of d,
$d_2 =$ remaining terms of d.

$a = (A/3)d = (A/3)d_1 + (A/3)d_2$
so $a - (A/3)d_2 = (A/3)d_1$.
Eliminate a: almost certainly
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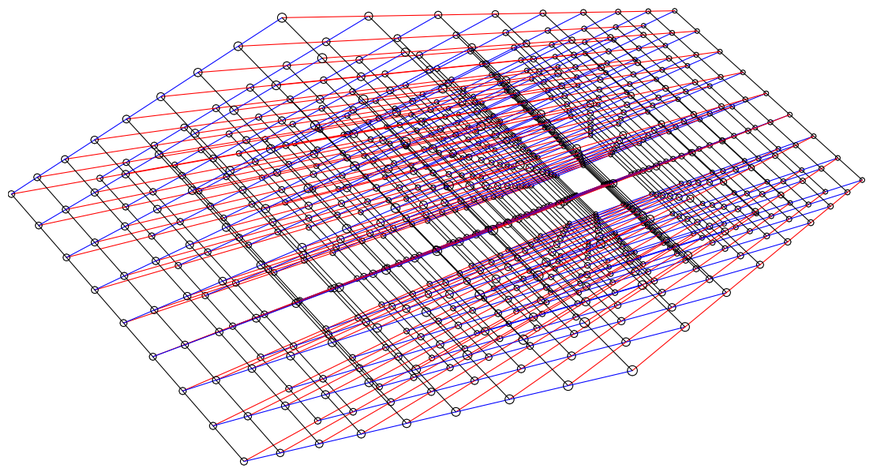

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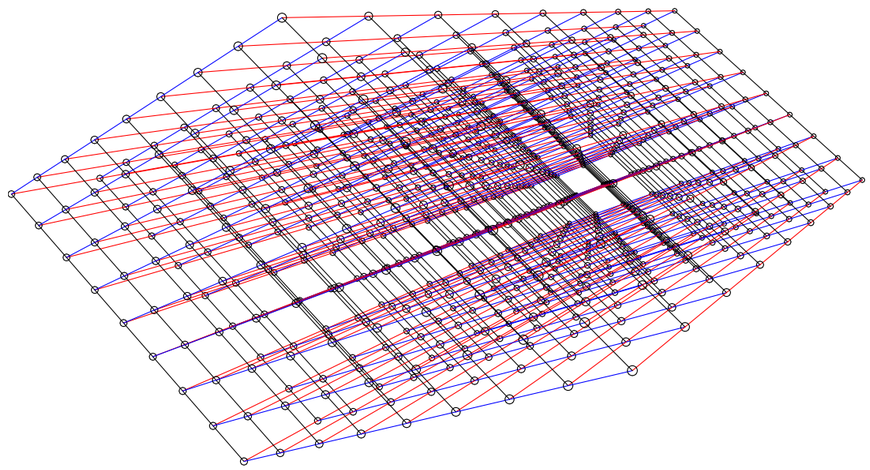

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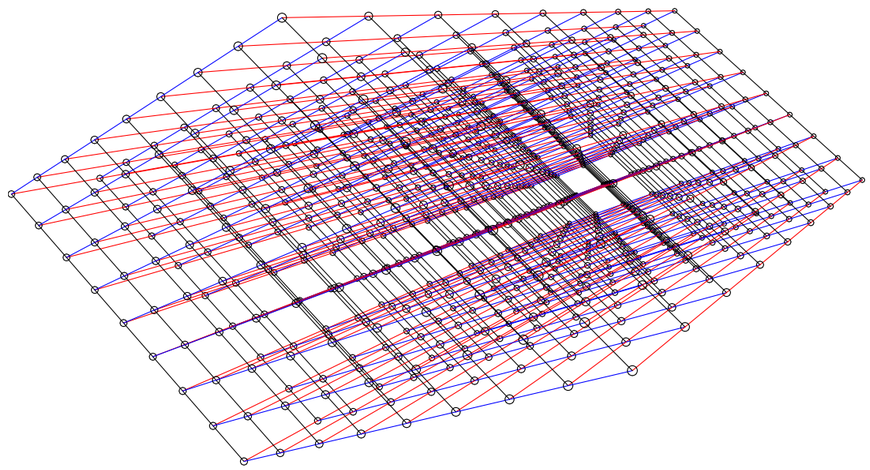

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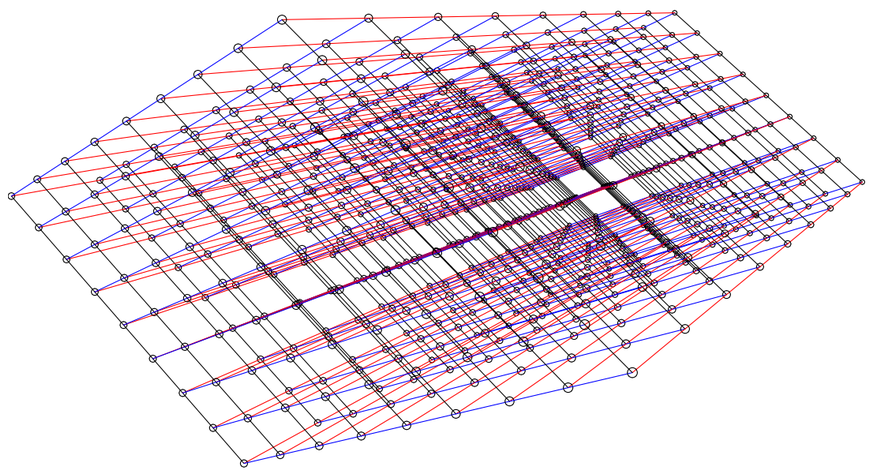

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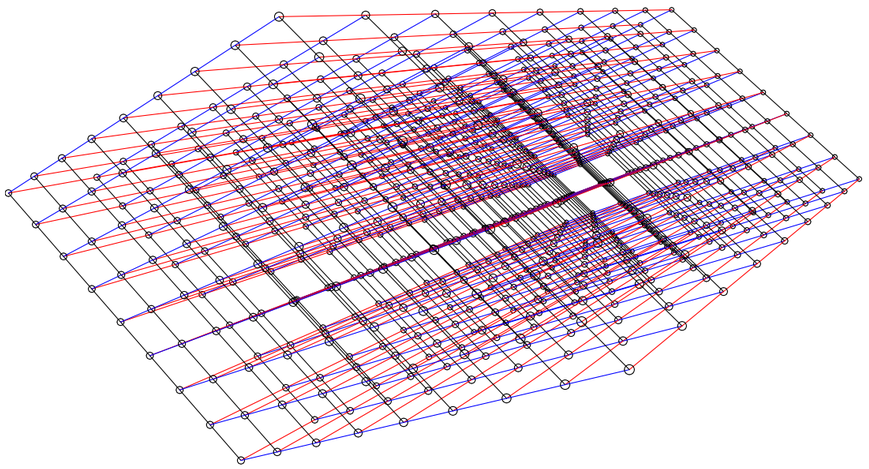

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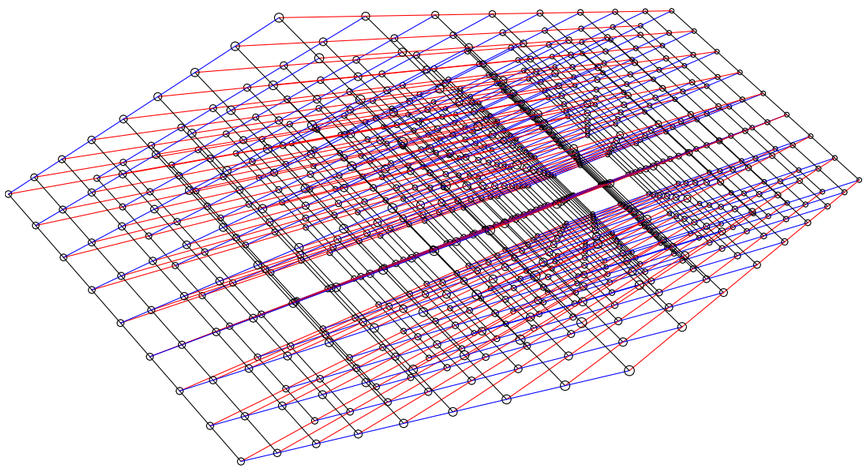

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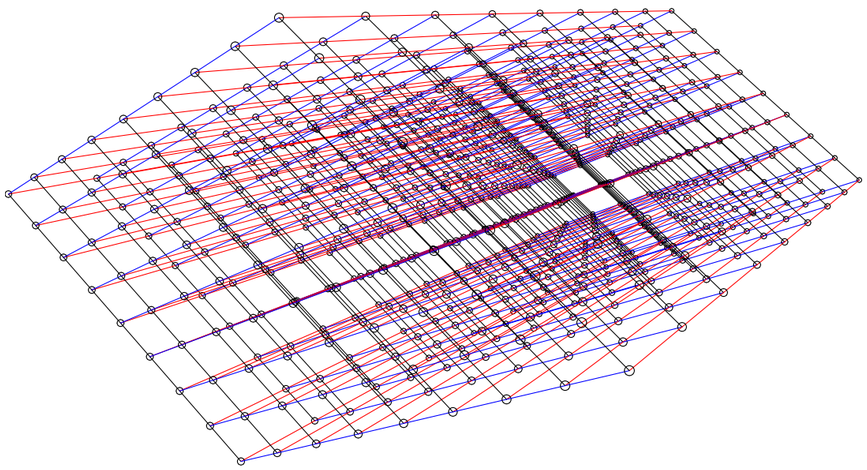

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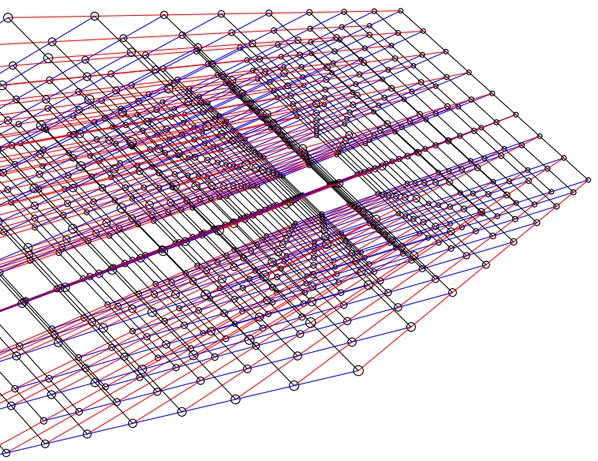

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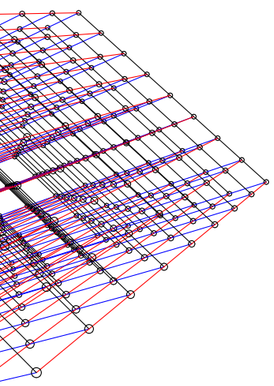

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Given public key A

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$d(A/3)$ is obtained from
$A/3, xA/3, \ldots, x^{n-1}A/3$
by a few additions, subtractions.

a is obtained from
$q, qx, qx^2, \ldots, qx^{n-1}$,
$A/3, xA/3, \ldots, x^{n-1}A/3$
by a few additions, subtractions.

ectors in lattices

$_1, b_2, \ldots, b_k \in \mathbf{Z}^n$,
shortest vector
$+ \ldots + \mathbf{Z}b_k$?

shortest nonzero vector?

orithm runs in poly time,
es a vector whose length
st $2^{n/2}$ times
f shortest nonzero vector.

algorithms (e.g., BKZ)
e shorter vectors
singly high speed.

Lattice view of NTRU

Given public key $A = 3a/d$.
Compute $A/3 = a/d$.

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(a, d) is
$(q, 0)$,
$(qx, 0)$,
$\vdots$
$(qx^{n-1},$
$(A/3, 1)$
$(xA/3, x$
$\vdots$
$(x^{n-1}A/$
by a few

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$v_k \in \mathbf{Z}^n$,
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$_k$?

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by a few additions, subtractions.

Write $A/3$ as
$H_0 + H_1x + \ldots + H_{n-1}x^{n-1}$.

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/ additions, subtractions.

is obtained from
$/3, \ldots, x^{n-1}A/3$
/ additions, subtractions.

ained from
$x^2, \ldots, qx^{n-1}$,
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is obtain
$(q, 0, \ldots$
$(0, q, \ldots$
$\vdots$
$(0, 0, \ldots$
$(H_0, H_1,$
$(H_{n-1}, H$
$\vdots$
$(H_1, H_2,$
by a few

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(a, d) is obtained from

$(q, 0)$,

$(qx, 0)$,

$\vdots$

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$(a_0, a_1, \ldots, a_{n-1}, c$

is obtained from

$(q, 0, \ldots, 0, 0, 0, \ldots$

$(0, q, \ldots, 0, 0, 0, \ldots$

$\vdots$

$(0, 0, \ldots, q, 0, 0, \ldots$

$(H_0, H_1, \ldots, H_{n-1}$

$(H_{n-1}, H_0, \ldots, H_n$

$\vdots$

$(H_1, H_2, \ldots, H_0, 0$

by a few additions

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(a, d) is obtained from
$(q, 0)$,
$(qx, 0)$,
$\vdots$
$(qx^{n-1}, 0)$,
$(A/3, 1)$,
$(xA/3, x)$,
$\vdots$
$(x^{n-1}A/3, x^{n-1})$
by a few additions, subtractions.

Write $A/3$ as
$H_0 + H_1x + \ldots + H_{n-1}x^{n-1}$.

48

$(a_0, a_1, \ldots, a_{n-1}, d_0, d_1, \ldots,$
is obtained from
$(q, 0, \ldots, 0, 0, 0, \ldots, 0)$,
$(0, q, \ldots, 0, 0, 0, \ldots, 0)$,
$\vdots$
$(0, 0, \ldots, q, 0, 0, \ldots, 0)$,
$(H_0, H_1, \ldots, H_{n-1}, 1, 0, \ldots,$
$(H_{n-1}, H_0, \ldots, H_{n-2}, 0, 1, \ldots$
$\vdots$
$(H_1, H_2, \ldots, H_0, 0, 0, \ldots, 1)$
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$(a_0, a_1, \ldots, a_{n-1}, d_0, d_1, \ldots, d_{n-1})$

is obtained from

$(q, 0, \ldots, 0, 0, 0, \ldots, 0)$,

$(0, q, \ldots, 0, 0, 0, \ldots, 0)$,

$\vdots$

$(0, 0, \ldots, q, 0, 0, \ldots, 0)$,

$(H_0, H_1, \ldots, H_{n-1}, 1, 0, \ldots, 0)$,

$(H_{n-1}, H_0, \ldots, H_{n-2}, 0, 1, \ldots, 0)$,

$\vdots$

$(H_1, H_2, \ldots, H_0, 0, 0, \ldots, 1)$

by a few additions, subtractions.

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$_1 x + \ldots + H_{n-1} x^{n-1}$.

$(a_0, a_1, \ldots, a_{n-1}, d_0, d_1, \ldots, d_{n-1})$
is obtained from
$(q, 0, \ldots, 0, 0, 0, \ldots, 0)$,
$(0, q, \ldots, 0, 0, 0, \ldots, 0)$,
$\vdots$
$(0, 0, \ldots, q, 0, 0, \ldots, 0)$,
$(H_0, H_1, \ldots, H_{n-1}, 1, 0, \ldots, 0)$,
$(H_{n-1}, H_0, \ldots, H_{n-2}, 0, 1, \ldots, 0)$,
$\vdots$
$(H_1, H_2, \ldots, H_0, 0, 0, \ldots, 1)$
by a few additions, subtractions.

$(a_0, a_1, \ldots$
is a surp
in lattice
$(q, 0, \ldots$

from

, subtractions.

$H_{n-1}x^{n-1}$.

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$(a_0, a_1, \ldots, a_{n-1}, d_0, d_1, \ldots, d_{n-1})$
is obtained from
$(q, 0, \ldots, 0, 0, 0, \ldots, 0)$,
$(0, q, \ldots, 0, 0, 0, \ldots, 0)$,
$\vdots$
$(0, 0, \ldots, q, 0, 0, \ldots, 0)$,
$(H_0, H_1, \ldots, H_{n-1}, 1, 0, \ldots, 0)$,
$(H_{n-1}, H_0, \ldots, H_{n-2}, 0, 1, \ldots, 0)$,
$\vdots$
$(H_1, H_2, \ldots, H_0, 0, 0, \ldots, 1)$
by a few additions, subtractions.

$(a_0, a_1, \ldots, a_{n-1}, c$
is a surprisingly sh
in lattice generate
$(q, 0, \ldots, 0, 0, 0, \ldots$

48

$(a_0, a_1, \ldots, a_{n-1}, d_0, d_1, \ldots, d_{n-1})$
is obtained from
$(q, 0, \ldots, 0, 0, 0, \ldots, 0)$,
$(0, q, \ldots, 0, 0, 0, \ldots, 0)$,
$\vdots$
$(0, 0, \ldots, q, 0, 0, \ldots, 0)$,
$(H_0, H_1, \ldots, H_{n-1}, 1, 0, \ldots, 0)$,
$(H_{n-1}, H_0, \ldots, H_{n-2}, 0, 1, \ldots, 0)$,
$\vdots$
$(H_1, H_2, \ldots, H_0, 0, 0, \ldots, 1)$
by a few additions, subtractions.

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$(a_0, a_1, \ldots, a_{n-1}, d_0, d_1, \ldots,$
is a surprisingly short vector
in lattice generated by
$(q, 0, \ldots, 0, 0, 0, \ldots, 0)$ etc.

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$(a_0, a_1, \ldots, a_{n-1}, d_0, d_1, \ldots, d_{n-1})$
is obtained from
$(q, 0, \ldots, 0, 0, 0, \ldots, 0)$,
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$\vdots$
$(0, 0, \ldots, q, 0, 0, \ldots, 0)$,
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Attacker searches for short vector
in this lattice using LLL etc.

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$(q, 0, \ldots, 0, 0, 0, \ldots, 0)$,
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1997 Coppersmith–Shamir
balancing: e.g., set up lattice
to contain $(10a, d)$
if d is chosen $10\times$ larger than a.

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$(q, 0, \ldots, 0, 0, 0, \ldots, 0)$,
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$\vdots$
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Exercise: Describe search for
(b, c) as a problem of finding
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$, 0, 0, 0, \ldots, 0)$,

$, q, 0, 0, \ldots, 0)$,

$\ldots, H_{n-1}, 1, 0, \ldots, 0)$,

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Quotient NTRU vs. product

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Bob sends $C = Ab + c$ in R_q.
Alice computes dC in R_q,
i.e., $3ab + dc$ in R_q.

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Alice reconstructs $3ab + dc$ in R,
using smallness of a, b, d, c.
Alice computes dc in R_3,
deduces c, deduces b.

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"Product NTRU"
2010 Lyubashevsk

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Alice generates A
for small random a

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Quotient NTRU vs. product NTRU

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"Product NTRU" (new nam
2010 Lyubashevsky–Peikert–

Everyone knows random $G \in$
Alice generates $A = aG + d$
for small random a, d.

Quotient NTRU vs. product NTRU

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Alice computes dC in R_q,
i.e., $3ab + dc$ in R_q.

Alice reconstructs $3ab + dc$ in R,
using smallness of a, b, d, c.
Alice computes dc in R_3,
deduces c, deduces b.

"Product NTRU" (new name),
2010 Lyubashevsky–Peikert–Regev:

Everyone knows random $G \in R_q$.
Alice generates $A = aG + d$ in R_q
for small random a, d.

Quotient NTRU vs. product NTRU

"Quotient NTRU" (new name)
is the structure we've seen:

Alice generates $A = 3a/d$ in R_q
for small random a, d:
i.e., $dA - 3a = 0$ in R_q.

Bob sends $C = Ab + c$ in R_q.
Alice computes dC in R_q,
i.e., $3ab + dc$ in R_q.

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"Product NTRU" (new name),
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Everyone knows random $G \in R_q$.
Alice generates $A = aG + d$ in R_q
for small random a, d.

Bob sends $B = Gb + e$ in R_q
and $C = m + Ab + c$ in R_q
where b, c, e are small and
each coefficient of m is 0 or $q/2$.

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Alice computes $C - aB$ in R_q,
i.e., $m + db + c - ae$ in R_q.
Alice reconstructs m,
using smallness of d, b, c, a, e.